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SWIFT POINTING AND THE ASSOCIATION BETWEEN GAMMA-RAY BURSTS AND GRAVITATIONAL WAVE BURSTS

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ABSTRACT

It is widely believed that gamma-ray bursts originate in relativistic fireballs produced by the merger or collapse of solar-mass compact objects. Gravitational waves should be associated with these violent, relativistic events, and their detection may shed light on the nature of the inner engine that powers the gamma-ray burst. Doing this requires joint observations of gamma-ray burst events with gravitational and gamma-ray detectors. Here we examine how the quality of an upper limit on the gravitational wave strength at Earth associated with gamma-ray burst observations depends on the relative orientation of the gamma-ray burst and gravitational wave detectors, and we apply our results to the particular case of the *Swift* Burst Alert Telescope and the LIGO gravitational wave detectors. A result of this investigation is a science-based "figure of merit" that can be used, together with other mission constraints, to optimize the pointing of the *Swift* telescope for the detection of gravitational waves associated with gamma-ray bursts.

Subject headings: gamma rays: bursts — gravitational waves

1. INTRODUCTION

It is widely believed that gamma-ray bursts (GRBs) are produced in intense relativistic fireballs generated by catastrophic events involving solar-mass compact objects (Cavallo & Rees 1978); see Mészáros (2002) for a recent review of GRB theories. Strong evidence exists that long-duration GRBs are associated with some core-collapse supernovae (e.g., Hjorth et al. 2003; Mészáros 2003; Price et al. 2003; Woosley 1993; Paczyński 1998; MacFadyen & Woosley 1999). Much less is known about short-duration GRBs, whose suggested progenitors include, for example, coalescing neutron star–neutron star or neutron star–black hole binaries (Paczyński 1986; Goodman 1986; Eichler et al. 1989; Mészáros & Rees 1997). These scenarios (though not all GRB models) are expected to result in the violent formation of a rapidly rotating solar-mass black hole surrounded by a debris torus. A relativistic ($\gamma \simeq 100$) fireball, powered perhaps by the release of binding energy as the debris torus is accreted onto the black hole (Woosley 1993), or by the spin energy of the black hole itself (Mészáros & Rees 1997), follows the black hole formation. The gamma-ray emission may take place at the site of crossing internal shock waves within the expanding relativistic fireball (Rees & Mészáros 1994) or as the fireball is decelerated by the interstellar medium (Mészáros & Rees 1993; Rees & Mészáros 1992).

Detection of the gravitational wave emissions from GRB systems may identify the nature of the elusive inner engine. A gravitational wave burst (GWB) is likely to be associated with the violent formation of the relativistic central engine that powers the fireball. Whereas the gamma rays are thought to be produced at distances $\geq 10^{13}$ cm from the central engine, these gravitational waves will be produced in or near the central engine, and thus could provide our most direct probe of it. For example, collapse or merger models lead to different GWB energies, spectra, and polarizations (Kobayashi & Mészáros 2003a, 2003b). Alternatively, gravitational wave production owing to toroidal instabilities in an accretion disk will be relatively long-lived and quasi-periodic (van Putten 2004), with an energy output several orders of magnitude higher than in the accretion mechanism proposed by Kobayashi & Mészáros (2003a, 2003b). In each case, the relative time of arrival of the GWB and GRB will depend on whether the GRB is generated by internal shocks in the exploding fireball or external shocks when the fireball is decelerated by the interstellar medium.[3] Observations of GWBs associated with GRBs may thus reveal details of the GRB mechanism that cannot be revealed through observations of the gamma rays alone. In this paper we examine how one can optimize GRB observations made in conjunction with gravitational wave

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[3] Although the fireball is highly relativistic, it travels subluminally; hence, the further the gamma rays are produced from the central engine the longer the time lag between the arrival of the gravitational waves and the gamma rays. This time lag can range from seconds to hundreds of seconds in the progenitor rest frame. On the other hand, the time lag between the formation of the central engine (which produces the GWB) and the launch of the relativistic jet (which produces the GRB) should be on the order of the dynamical timescale of milliseconds and is a much smaller effect.

observations for the purpose of detecting an association between GRBs and GWBs, a first step in the ambitious program of developing gravitational wave observations into a tool of astronomical discovery.

Finn, Mohanty, & Romano (1999) have described how the cross-correlated output of two gravitational wave detectors, taken in coincidence with GRBs, can be used to detect or place upper limits on the emission of GWBs by GRB systems. This procedure has already been used in the analysis of data from the Explorer and Nautilus gravitational wave detectors at times associated with 47 GRBs detected by the *BeppoSAX* satellite (Astone et al. 2002). The authors bound, at 95% confidence, the rms gravitational wave strain $h_{\rm rms}$ associated with these GRBs to less than 6.5×10^{-19} in the gravitational wave detector waveband (~0.5 Hz about 900 Hz), assuming that the gamma rays originate in internal shocks. Finn et al. (1999) estimate that 1000 GRB observations combined with observations from the (broadband) initial LIGO detectors (Sigg 2001) could produce an upper limit on the gravitational wave strain associated with GRBs of approximately $h_{\rm rms} \leq 1.7 \times 10^{-22}$ at 95% confidence.

An important consequence of the cosmological origin of GRBs is their isotropic distribution on the sky. In their original work Finn et al. (1999) assumed that GRBs would be detected isotropically as well, i.e., that the GRB detector had an isotropic antenna pattern. They did note, however, that the *Swift* satellite,[4] a next-generation multiwavelength satellite dedicated to the study of GRBs, does not have an isotropic antenna pattern and that this has potentially important consequences for the ability of the combined GRB/gravitational wave detector array to detect or limit the gravitational wave flux on Earth owing to GRBs. Specifically, since the gravitational wave and GRB detectors have differential sensitivity on the sky, it is advantageous to arrange, whenever possible, that gravitational wave and GRB detectors used together are "pointed" in the same direction. Here we study this question specifically in the context of the *Swift* satellite and the LIGO gravitational wave detectors; i.e., we determine, as a function of *Swift*'s pointing, the sensitivity of the *Swift*/LIGO detector array to gravitational waves from GRBs detected by *Swift*. We propose a figure of merit that can be used in *Swift* mission scheduling, suitably weighted together with other mission science goals, to optimize the sensitivity of the *Swift*/LIGO array for detecting the gravitational wave flux from GRBs. We find that the upper limit that can be placed on the mean-square gravitational wave amplitude differs by a factor of 4 between best and worst orientations of the satellite.

In § 2 we review the technique proposed by Finn et al. (1999) for detecting or placing an upper limit on the gravitational wave strength associated with GRBs. We extract the direction dependence of this upper limit in § 3, and evaluate it for the case of the LIGO detectors and the *Swift* Burst Alert Telescope (BAT) in § 4. We conclude with some brief remarks in § 5.

2. OBSERVING A GRB-GWB ASSOCIATION

Finn et al. (1999) described how the signals from two independent gravitational wave detectors can be analyzed to identify the gravitational wave signal associated with GRBs and either bound or measure the population average of the gravitational wave flux on Earth from this potential source. The key idea is to compare the mean correlated output of the detectors during GRB events ("on" times) to the mean correlated output when no GRB is detected ("off" times). Since gravitational waves from GRBs would produce small correlations in the output of the two detectors, a statistically significant difference in the mean correlated output between on and off times would constitute an indirect detection of GWBs from GRBs (Finn and coworkers use the term "association"). Alternately, the absence of a statistically significant difference would allow one to set an upper limit on the strength of any gravitational waves associated with GRBs.

The analysis proposed by Finn et al. (1999) is inherently statistical in that it aims to detect or set upper limits on the gravitational wave strength at Earth due to an ensemble of GRB systems. This approach is motivated by the fact that the direct detection of an individual GWB from a GRB system using the present generation of gravitational wave detectors is difficult. For example, Frail et al. (2001) estimate the true rate of GRB-supernova events at 1 per year within a distance of 100 Mpc; Kobayashi & Mészáros (2003b) estimate that individual sources at these distances would be marginally detectable for the advanced LIGO detectors and hence are unlikely to be detected individually by the initial LIGO detectors.

In this section we review the analysis methodology of Finn et al. (1999) in anticipation of using it to determine the sensitivity of joint LIGO/*Swift* observations to the detectors' relative orientation.

2.1. *Detecting a GRB-GWB Association*

Consider a set of N GRB detections. Assume that, as a result of each detection, we know the direction to the source $\hat{\boldsymbol{\Omega}}_k$ and the arrival time τ_k of the burst at Earth's barycenter. For our purposes, each GRB observation is completely characterized by the pair $(\hat{\boldsymbol{\Omega}}_k,\ \tau_k)$.[5] Focus attention on a pair of gravitational wave detectors located at positions $\boldsymbol{D}_i$ $(i = 1,\ 2)$ relative to Earth's barycenter. The arrival time of GRB k at detector i is

$$t_k^{(i)} = \tau_k - \hat{\boldsymbol{\Omega}}_k \cdot \boldsymbol{D}_i, \tag{1}$$

in units in which the speed of light c is unity.

The two LIGO detectors are very nearly coplanar and coaligned. Consequently, a plane gravitational wave incident on the detector pair from the direction $\hat{\boldsymbol{\Omega}}_k$ will lead to correlated detector responses with a time lag equal to

$$\Delta t_k = t_k^{(2)} - t_k^{(1)}. \tag{2}$$

To identify the presence of GWBs associated with GRBs, Finn et al. (1999) focus attention on the correlated energy in the detector outputs corresponding to plane GWBs incident on the detectors from the direction of the corresponding GRBs, i.e., the correlation of the detector outputs at times that differ by Δt_k.

Let $s_i(t)$ be the calibrated output of gravitational wave detector $\boldsymbol{D}_i$, which we assume to consist of detector noise $n_i(t)$

[4] See http://swift.gsfc.nasa.gov.

[5] Afterglow observations will also give the distance to the GRB source (Metzger et al. 1997), but this information is not used in the proposed analysis.

and a possible gravitational wave signal $h_i(t)$ *produced by the GRB source*:

$$s_i(t) = n_i(t) + h_i(t). \tag{3}$$

Finn et al. (1999) define

$$S\left(\hat{\boldsymbol{\Omega}}_k,\ \tau_k\right) = \int_0^T dt \int_0^T dt'\, s_1\left(t_k^{(1)} - t\right) Q(t - t') s_2\left(t_k^{(2)} - t'\right), \tag{4}$$

as a measure of the cross-correlation of the two detectors corresponding to the GRB characterized by $(\hat{\boldsymbol{\Omega}}_k,\ \tau_k)$. Here Q is a freely specifiable, symmetric filter function, and T is chosen large enough to encompass the range of possible times by which the gravitational waves from a GRB event may precede the gamma rays, which is typically thought to be of order 1 s for GRBs produced by internal shocks and 100 s for GRBs produced by external shocks (Sari & Piran 1997; Kobayashi, Piran, & Sari 1997).

Writing the detector output s_i as the sum of the detector noise n_i and the gravitational wave signal h_i associated with the GRB, we can write

$$S_k \equiv S(\hat{\boldsymbol{\Omega}}_k,\ \tau_k) = \langle n_1,\ n_2\rangle + \langle n_1,\ h_2\rangle + \langle h_1,\ n_2\rangle + \langle h_1,\ h_2\rangle, \tag{5}$$

where

$$\langle f,\ g\rangle = \int_0^T dt \int_0^T dt'\, f\left(t_k^1 - t\right) Q(t - t') g\left(t_k^2 - t'\right). \tag{6}$$

The terms $\langle n_i,\ h_j\rangle$ in equation (5) vanish in the mean over an ensemble of noise, since the noise in our gravitational wave detector is uncorrelated with any gravitational wave signal. The term $\langle n_1,\ n_2\rangle$ is, in the noise ensemble mean, a constant, which will be zero if the noise in the two detectors is uncorrelated.

All four of the contributions to S in equation (5) are generally unknown for any particular GRB. Correspondingly, Finn et al. (1999) consider the collection $\hat{S}_{\rm on}$ of S_k,

$$\hat{S}_{\rm on} = \{S_k : k = 1 \ldots N\}, \tag{7}$$

and a second collection $\hat{S}_{\rm off}$

$$\hat{S}_{\rm off} = \left\{S'_m : m = 1 \ldots M\right\}, \tag{8}$$

where each S'_m is constructed as in equation (4) but with a $(\boldsymbol{\Omega},\ \tau)$ pair chosen randomly and not associated with a GRB. The collections $\hat{S}_{\rm on}$ and $\hat{S}_{\rm off}$ are samples drawn from *populations* $S_{\rm on}$ and $S_{\rm off}$. The sample means $\hat{\mu}_{\rm on}$ and $\hat{\mu}_{\rm off}$ and variances $\hat{\sigma}^2_{\rm on}$ and $\hat{\sigma}^2_{\rm off}$ are estimates of the *population means* $\mu_{\rm on}$ and $\mu_{\rm off}$ and variances $\sigma^2_{\rm on}$ and $\sigma^2_{\rm off}$. These are, in turn, related by

$$\mu_{\rm on} - \mu_{\rm off} = \overline{\langle h_1,\ h_2\rangle}, \tag{9}$$

$$\sigma^2_{\rm on} - \sigma^2_{\rm off} = \overline{\langle n_1,\ h_2\rangle^2} + \overline{\langle n_2,\ h_1\rangle^2} + O\left(h^3\right), \tag{10}$$

where the overbar represents a mean over the population of GRBs. Comparing the two sample sets $\hat{S}_{\rm on}$ and $\hat{S}_{\rm off}$ thus provides a sensitive measure of the presence or absence of gravitational waves associated with GRBs.

Note that there may be a flux of gravitational waves from other sources incident coincidentally on the detector at the same time, including GWBs from GRB events that are not observed by *Swift* (Frail et al. 2001). Since this radiation is not correlated with the observed GRBs it will make equal contributions to the mean on- and off-source cross-correlation measurements (7) and (8) and so can be ignored. The net effect of such unseen signals is merely to increase the expected variance in the cross-correlations.

When the detector noise is sufficiently well-behaved that terms like $\langle n_1,\ h_2\rangle$, $\langle n_2,\ h_1\rangle$, and $\langle n_1,\ n_2\rangle$ are normally distributed, Student's t-test (Snedecor & Cochran 1967) can be used to distinguish between the two sample sets; in other cases a nonparametric test such as the Mann-Whitney test (Siegal & Castellan 1988) can be used.

In Student's t-test the difference between the two distributions represented by the sample sets $\hat{S}_{\rm on}$ and $\hat{S}_{\rm off}$ is characterized by the t-statistic:

$$\hat{t} = \frac{\hat{\mu}_{\rm on} - \hat{\mu}_{\rm off}}{\hat{\Sigma}} \sqrt{\frac{N_{\rm on} N_{\rm off}}{N_{\rm on} + N_{\rm off}}}, \tag{11}$$

$$\hat{\Sigma}^2 = \frac{\hat{\sigma}^2_{\rm on}(N_{\rm on} - 1) + \hat{\sigma}^2_{\rm off}(N_{\rm off} - 1)}{N_{\rm on} + N_{\rm off} - 2}, \tag{12}$$

If both $\hat{S}_{\rm on}$ and $\hat{S}_{\rm off}$ are drawn from the same normal distribution, then the distribution of $\hat{t}$ is given by Student's distribution with $N_{\rm on} + N_{\rm off} - 2$ degrees of freedom (Cramer 1999). This distribution itself tends toward a normal distribution with unit variance when $N_{\rm on} + N_{\rm off}$ is large.

Now suppose that there is no GWB-GRB association. In this event $\hat{S}_{\rm on}$ and $\hat{S}_{\rm off}$ are drawn from the same distribution and there is a number $t_0(p)$ such that $\hat{t}$ will be less than $t_0(p)$ in a fraction p of all observations of sample sets $\hat{S}_{\rm on}$ and $\hat{S}_{\rm off}$.[6] If, in our particular observation of $\hat{S}_{\rm on}$ and $\hat{S}_{\rm off}$, $\hat{t}$ is less than $t_0(p)$, then we accept the hypothesis that there are *no* gravitational waves associated with GRBs. If, on the other hand, we find $\hat{t}$ greater than $t_0(p)$, then we reject, with confidence p, this hypothesis; i.e., we assert that there is an association of gravitational waves with GRBs.

2.2. *Setting an Upper Limit on the Gravitational Wave Strength Associated with GRBs*

As described in the previous section, Student's t-test tells us only whether there is a link between GWBs and GRBs. An alternative analysis, also described by Finn et al. (1999), uses the t-statistic to derive a confidence interval or upper limit on the population-averaged gravitational wave flux associated with GRBs from a measured value $\hat{t}$ of the t-statistic. In this analysis we derive the classical confidence interval or upper limit from the probability distribution $P(t|h,\ I)$ of the t-statistic assuming that GRBs radiate GWBs with intrinsic amplitude described by h and other model parameters (GWB luminosity function, burst characteristic, etc.) described by I.

In the limit that the GWBs are weak relative to the sensitivity of the individual gravitational wave detectors and the numbers of on- and off-source observations are separately large, Finn et al. (1999) showed that

$$P(t|h,\ I) = N\left(t\middle|\mu,\ \frac{\sigma^2}{2}\right), \tag{13}$$

[6] On physical grounds, the expectation value of $\hat{t}$ will be positive semidefinite for the LIGO detector pair if gravitational waves are associated with GRBs.

where

$$\mu = \overline{\langle h_1,\ h_2\rangle} \tag{14}$$

$$\sigma^2 = \frac{T}{4}\int_{-\infty}^{\infty} df\, P_1(f)P_2(f)\left|\tilde{Q}(f)\right|^2. \tag{15}$$

Here $P_i(f)$ is the one-sided power spectral density of the ith detector, defined as

$$P_i(|f|) = 2\int_{-\infty}^{\infty} dt\, e^{i2\pi ft} n_i(\tau)n_i(\tau + t), \tag{16}$$

$\overline{\langle h_1,\ h_2\rangle}$ is the average of $\langle h_1,\ h_2\rangle$ over the GRB population and the associated GWB luminosity function described by I, $N(t|\mu,\ \nu)$ is the normal distribution with mean μ and variance ν, and $\tilde{Q}(f)$ is the Fourier transform of $Q(\tau)$. For larger amplitude GRBs or different sample sizes (e.g., smaller number of GRB observations), the distribution can be determined via Monte Carlo simulations. An observation of t thus allows us to find a confidence limit on $\overline{\langle h_1,\ h_2\rangle}$ (Feldman & Cousins 1998), which describes the flux of gravitational waves at Earth owing to GRBs.

The upper limit clearly gets better if we observe more GRBs, i.e., if $N_{\rm on}$ is as large as possible. An estimate of the number of GRB observations required to set a given upper limit on the amplitude of the gravitational waves is given in Finn et al. (1999). They find that the upper limit on the mean-square signal power $h^2_{\rm rms}$ is proportional to $N_{\rm on}^{-1/2}$; they estimate the attainable 95% upper limit on $h^2_{\rm rms}$ in a frequency band of 100 Hz as[7]

$$h^2_{\rm rms} \le \left(4.8\times 10^{-22}\right)^2 \frac{S_0}{\left(3\times 10^{-23}\ {\rm Hz}^{-1/2}\right)^2}\left(\frac{T}{0.5s}\frac{100}{N_{\rm on}}\right)^{1/2}. \tag{17}$$

Here S_0 is the one-sided power spectral density of the noise in the detectors and T is as in equation (4). Therefore 100 GRBs should be sensitive to $h_{\rm rms}$ of approximately 4.8×10^{-22}; this may be compared to the estimates of Kobayashi & Mészáros (2003b) for the gravitational wave amplitudes from various GRB progenitors.

3. DIRECTION DEPENDENCE OF THE UPPER LIMIT

The analysis described in Finn et al. (1999) involved two gravitational wave detectors. It assumed for simplicity that these two detectors had identical isotropic antenna patterns and that each was sensitive to exactly the same gravitational wave polarization. Here we relax all of these approximations; i.e., we properly account for the position and orientation of the gravitational wave detectors, focusing particularly on the two LIGO detectors on the Earth and the dependence of their sensitivity to the direction to the GRB source. Our result is an expression for the dependence of the upper limit on population-averaged gravitational wave strength $\overline{\langle h_1,\ h_2\rangle}$ as a function of the distribution of *detected* GRBs on the sky. In § 4 we combine this result with the directional sensitivity of the *Swift* detector to determine the dependence on *Swift* pointing of the upper limit on $\overline{\langle h_1,\ h_2\rangle}$ that can be set by joint LIGO/*Swift* observations.

The gravitational wave contribution h_i to the output of the ith LIGO detector is, in the small-antenna limit,[8] a linear function of the physical gravitational wave strain $h_{ab}(t,\ \boldsymbol{x})$,

$$h_i(t) = h_{ab}(t,\ \boldsymbol{x}_i)d_i^{ab}, \tag{18}$$

where $\boldsymbol{x}_i$ is the gravitational wave detector's location. For interferometer i with arms pointing in the directions given by unit vectors $\hat{\boldsymbol{X}}_i$, $\hat{\boldsymbol{Y}}_i$,

$$d_i^{ab} = \frac{1}{2}\left(X_i^a X_i^b - Y_i^a Y_i^b\right). \tag{19}$$

[With this normalization, $h_i(t)$ is equal to the fractional change in differential arm length.] For GWBs incident on the Earth from direction $\hat{\boldsymbol{\Omega}}$,

$$h_{ab}\left(t_k^{(1)} - t,\ \boldsymbol{x}_1\right) = h_{ab}\left(t_k^{(2)} - t,\ \boldsymbol{x}_2\right) = h_{ab}(\tau_k - t,\ \boldsymbol{0}) \tag{20}$$

where $t_k^{(i)}$ is defined in terms of the direction to the source $\hat{\boldsymbol{\Omega}}$ by equation (1). We can thus ignore the physical separation of the detectors when computing the cross-correlation statistic (cf. eq. [4]). Finally, it is convenient to resolve h_{ab} on the two polarization tensors $\boldsymbol{\epsilon}^+$ and $\boldsymbol{\epsilon}^\times$,

$$h_{ab}\left(t,\ \hat{\boldsymbol{\Omega}}\right) = h_+(t)\epsilon^+_{ab}\left(\hat{\boldsymbol{\Omega}}\right) + h_\times(t)\epsilon^\times_{ab}\left(\hat{\boldsymbol{\Omega}}\right), \tag{21}$$

where

$$\boldsymbol{\epsilon}^+ : \boldsymbol{\epsilon}^+ = \boldsymbol{\epsilon}^\times : \boldsymbol{\epsilon}^\times = 2, \tag{22}$$

$$\boldsymbol{\epsilon}^+ : \boldsymbol{\epsilon}^\times = 0, \tag{23}$$

and

$$\boldsymbol{\epsilon}^+ \cdot \hat{\boldsymbol{\Omega}} = \boldsymbol{\epsilon}^\times \cdot \hat{\boldsymbol{\Omega}} = 0. \tag{24}$$

Lacking any detailed model for the gravitational waves that may be produced in a GRB event, we make the following assumptions about h_+ and $h_\times$:

1. The waves have equal power in the two polarizations:

$$\overline{h_+(t)h_+(t')} = \overline{h_\times(t)h_\times(t')}. \tag{25}$$

2. The two polarizations are uncorrelated:

$$\overline{h_+(t)h_\times(t')} = 0. \tag{26}$$

Focus attention now on the mean gravitational wave contribution $\langle h_1,\ h_2\rangle$ to the t-statistic. This contribution depends on both the gravitational wave detector sensitivity to GWBs arriving from different directions, as well as the GRB detector sensitivity to GRBs from different directions (which determines the relative number of bursts that will be observed from that direction). For GWBs arriving from direction $\hat{\boldsymbol{\Omega}}$,

$$\langle h_1,\ h_2\rangle = \rho_{\rm GWB}\left(\hat{\boldsymbol{\Omega}}\middle|\mathbf{d}_1,\ \mathbf{d}_2\right)\int_0^T dt\int_0^T dt'\, Q(t-t')h_+(t)h_+(t'), \tag{27}$$

[7] In quoting the result of Finn et al. (1999) we include an additional factor of 5/2 in $h^2_{\rm rms}$ to account for the sky-averaged LIGO antenna pattern function $\rho_{\rm GWB}$ introduced in § 3.

[8] This is appropriate for gravitational wave frequencies in the LIGO detector band.

where

$$\rho_{\mathrm{GWB}}\left(\hat{\boldsymbol{\Omega}}\middle|\mathbf{d}_1,\ \mathbf{d}_2\right) \equiv \sum_{A=+,\times}\left[\mathbf{d}_1 : \boldsymbol{\epsilon}^A\left(\hat{\boldsymbol{\Omega}}\right)\right]\left[\mathbf{d}_2 : \boldsymbol{\epsilon}^A\left(\hat{\boldsymbol{\Omega}}\right)\right] \tag{28}$$

describes the direction dependence of the sensitivity of the gravitational wave detector pair to the GWB (cf. eqs. [4], [25], and [26]).

To complete the evaluation of $\overline{\langle h_1,\ h_2\rangle}$ consider the fraction of GRB detections that arise from different patches on the sky. Since the intrinsic GRB population is isotropic, the distribution of detections on the sky depends entirely on the directional sensitivity of the GRB detector. Let the fraction of GRB detections in a sky patch of area $d^2\hat{\boldsymbol{\Omega}}$ centered at $\hat{\boldsymbol{\Omega}}$ be given by

$$\rho_{\mathrm{GRB}}\left(\hat{\boldsymbol{\Omega}}\middle|\hat{\boldsymbol{\Omega}}',\ \hat{\boldsymbol{n}}'\right) d^2\hat{\boldsymbol{\Omega}}, \tag{29}$$

where the GRB detector orientation is given by $\hat{\boldsymbol{\Omega}}'$, the direction in which the detector is pointed, and $\hat{\boldsymbol{n}}'$, which describes the rotation of the satellite about its pointing direction.[9]

In terms of ρ_{GWB} and ρ_{GRB} the mean gravitational wave contribution $\overline{\langle h_1,\ h_2\rangle}$ to the t-statistic for a GRB detector with fixed orientation $\left(\hat{\boldsymbol{\Omega}}',\ \hat{\boldsymbol{n}}'\right)$ is thus

$$\begin{aligned}\overline{\langle h_1,\ h_2\rangle} = &\left[\int d^2\hat{\boldsymbol{\Omega}}\,\rho_{\mathrm{GWB}}\left(\hat{\boldsymbol{\Omega}}\middle|\mathbf{d}_1,\ \mathbf{d}_2\right)\rho_{\mathrm{GRB}}\left(\hat{\boldsymbol{\Omega}}\middle|\hat{\boldsymbol{\Omega}}',\ \hat{\boldsymbol{n}}'\right)\right]\\ &\times\left[\int_0^T dt' \int_0^T dt\, Q(t-t')h_+(t)h_+(t')\right].\end{aligned} \tag{30}$$

The first bracketed term contains all the direction and orientation dependence of the gravitational wave and gamma-ray burst detectors, while the second term is strictly a property of the gravitational waves without reference to the orientation of the detectors. Correspondingly, the upper limit on the gravitational wave strength averaged over the observed GRB population when the orientation of GWB and the GRB detectors are given by $\left(\hat{\boldsymbol{\Omega}}',\ \hat{\boldsymbol{n}}',\ \mathbf{d}_1,\ \mathbf{d}_2\right)$ is inversely proportional to

$$\zeta\left(\hat{\boldsymbol{\Omega}}',\ \hat{\boldsymbol{n}}',\ \mathbf{d}_1,\ \mathbf{d}_2\right) \equiv \int d^2\hat{\boldsymbol{\Omega}}\,\rho_{\mathrm{GRB}}\left(\hat{\boldsymbol{\Omega}}\middle|\hat{\boldsymbol{\Omega}}',\ \hat{\boldsymbol{n}}'\right)\rho_{\mathrm{GWB}}\left(\hat{\boldsymbol{\Omega}}\middle|\mathbf{d}_1,\ \mathbf{d}_2\right). \tag{31}$$

Satellite GRB detectors orbit the Earth and so their orientation is constantly changing; similarly, the orientation of Earth-based gravitational wave detectors are constantly changing as the Earth rotates about its axis. The quantity $\overline{\langle h_1,\ h_2\rangle}$ will, in the end, involve the time average of ζ over all these motions. Since our principal purpose here is to evaluate the sensitivity of the GRB/GWB detector array to GWBs from GRBs as a function of the relative orientations of the detectors we focus on ζ.

Clearly ζ can be regarded as a figure of merit that describes how capable the gravitational wave/gamma-ray burst detector combination is at identifying GWBs associated with GRBs as a function of the detector orientations. This figure of merit may be normalized to have a maximum of unity; however, regardless of the normalization

$$\left[\zeta\left(\hat{\boldsymbol{\Omega}}',\ \hat{\boldsymbol{n}}',\ \mathbf{d}_1,\ \mathbf{d}_2\right)/\zeta\left(\hat{\boldsymbol{\Omega}}'',\ \hat{\boldsymbol{n}}'',\ \mathbf{d}_1,\ \mathbf{d}_2\right)\right]^{-1} \tag{32}$$

is the ratio of the upper limits on the mean-square gravitational wave amplitude that can be attained by orienting the GRB satellite as $\left(\hat{\boldsymbol{\Omega}}',\ \hat{\boldsymbol{n}}'\right)$ versus $\left(\hat{\boldsymbol{\Omega}}'',\ \hat{\boldsymbol{n}}''\right)$. To the extent that, e.g., the GRB detector orientation $\left(\hat{\boldsymbol{\Omega}}',\ \hat{\boldsymbol{n}}'\right)$ can be manipulated on orbit, choosing orientations that maximize ζ will lead to larger signal contributions to the t-statistic and thus more sensitive measurements of the gravitational wave strength associated with GRBs.

4. LIGO AND *SWIFT*

Let us now consider the special case of the BAT on the *Swift* satellite,[10] which is scheduled for launch in late spring 2004, and the LIGO gravitational wave detectors (Sigg 2001). The BAT is a wide field-of-view coded-aperture gamma-ray imager that will detect and locate GRBs with arc-minute positional accuracy. Its sensitivity to GRBs depends on the angle λ between the line of sight to the GRB and the BAT axis, as well as the rotational orientation of the satellite about the BAT axis. The BAT sensitivity averaged over the azimuthal angle as a function of λ has been evaluated by the BAT instrument team. It has the approximate form[11]

$$\rho_{\mathrm{GRB}} = \begin{cases} 2\cos\lambda - 1 + 0.077\sin[13(1-\cos\lambda)] & \lambda \in [0,\ \pi/3], \\ 0 & \text{otherwise.}\end{cases} \tag{33}$$

The azimuthal-averaged sensitivity drops to half its maximum value for $\lambda \approx 40°$ and goes to zero at $\lambda \approx 60°$. For the purposes of illustration we use this expression for the BAT sensitivity.

The second component of ζ is the function ρ_{GWB}, which depends on the projection of the gravitational wave strain h_{ab} on to the gravitational wave detector (cf. eqs. [18] and [21]). To evaluate ρ_{GWB}, introduce an Earth-centered Cartesian coordinate system, described by unit basis vectors $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, and $\hat{\boldsymbol{z}}$, with $\hat{\boldsymbol{z}}$ pointing parallel to the Earth's polar axis and in the direction of the north celestial pole, $\hat{\boldsymbol{x}}$ parallel to the line that runs in the equatorial plane from Earth's center to the intersection of the equator with the prime meridian at Greenwich, and $\hat{\boldsymbol{y}}$ chosen to form a right-handed coordinate system. Similarly, we introduce the usual spherical-polar coordinate system,

$$r^2 = x^2 + y^2 + z^2, \tag{34}$$

$$\cos\theta = z/r, \tag{35}$$

$$\tan\phi = y/x. \tag{36}$$

In these coordinates we write the gravitational wave polarization vectors as

$$\epsilon^+_{ab} = \hat{m}_a\hat{m}_b - \hat{n}_a\hat{n}_b, \tag{37}$$

$$\epsilon^\times_{ab} = \hat{m}_a\hat{n}_b + \hat{n}_a\hat{m}_b, \tag{38}$$

where[12]

$$\hat{\boldsymbol{m}} = -\hat{\boldsymbol{x}}\sin\phi + \hat{\boldsymbol{y}}\cos\phi, \tag{39}$$

$$\hat{\boldsymbol{n}} = \hat{\boldsymbol{x}}\cos\phi\cos\theta + \hat{\boldsymbol{y}}\sin\phi\cos\theta - \hat{\boldsymbol{z}}\sin\theta, \tag{40}$$

$$\hat{\boldsymbol{\Omega}} = \hat{\boldsymbol{x}}\cos\phi\sin\theta + \hat{\boldsymbol{y}}\sin\phi\sin\theta + \hat{\boldsymbol{z}}\cos\theta. \tag{41}$$

[9] The detailed form of ρ_{GRB} depends on the detailed construction of the GRB detector. We consider the case of the *Swift* BAT below.

[10] See http://swift.gsfc.nasa.gov.

[11] See the BAT section of the *Swift* Web site, http://swift.gsfc.nasa.gov/science/instruments/bat.html

[12] Our conventions follow those of Allen & Romano (1999), except that we use $\hat{\boldsymbol{\Omega}}$ to denote the direction *to* the GRB/GWB source on the sky, rather than the propagation direction of the GWB. The net effect is $\hat{\boldsymbol{m}} \to -\hat{\boldsymbol{m}}$ in eq. (39).

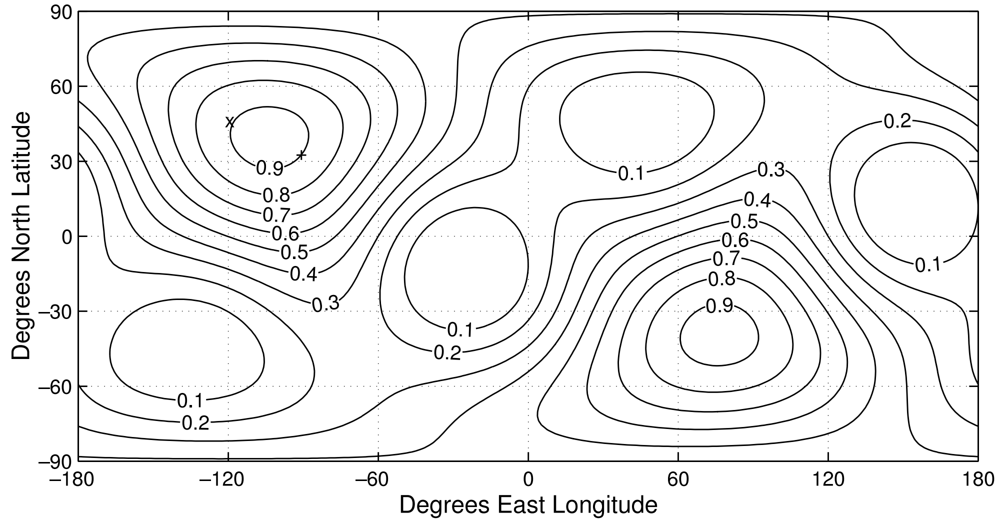

FIG. 1.—LIGO sensitivity pattern $\rho_{\rm GWB}$ (eq. [28]) in Earth-based coordinates for GWBs satisfying eqs. (25) and (26). The plus sign and cross mark the locations of the LLO and LHO detectors, respectively. The array is most sensitive in the directions orthogonal to the plane of the LIGO detectors, corresponding to the two peaks at upper left and lower right. The sensitivity is lowest in the plane of the detectors near the directions at 45° to the arms and vanishes in certain directions, producing the four wells of low sensitivity. The sensitivity has been scaled to range over [0, 1] in this plot.

Similarly, denoting the detector projection tensor (cf. eq. [19]) for the LIGO Hanford (LIGO Livingston) Observatory detector by $\mathbf{d}_{\rm LHO}$ ($\mathbf{d}_{\rm LLO}$), we have (Althouse et al. 2001)

$$\mathbf{d}_{\rm LHO} = \begin{pmatrix} -0.3926 & -0.0776 & -0.2474 \\ -0.0776 & 0.3195 & 0.2280 \\ -0.2474 & 0.2280 & 0.0731 \end{pmatrix}, \quad (42)$$

$$\mathbf{d}_{\rm LLO} = \begin{pmatrix} 0.4113 & 0.1402 & 0.2473 \\ 0.1402 & -0.1090 & -0.1816 \\ 0.2473 & -0.1816 & -0.3022 \end{pmatrix}. \quad (43)$$

Figure 1 shows the antenna pattern $\rho_{\rm GWB}$ of equation (28). Since the two LIGO detectors share nearly the same plane and have arms nearly aligned with each other, their combined antenna pattern is very similar to that for a single interferometer. In particular, the LHO/LLO detector combination is most sensitive to radiation arriving from a direction orthogonal to the (nearly common) plane of the detector arms (corresponding to the two peaks in Fig. 1) and least sensitive to radiation arriving in the detector arm plane and parallel to the (nearly common) arm bisector (producing the four wells of low sensitivity in Fig. 1). It is also precisely zero in certain directions.

Convolving $\rho_{\rm GWB}$ with the *Swift* sensitivity function ρ_{Swift} as in equation (31) gives the figure of merit ζ for the *Swift* pointing, which is shown in Figure 2. This plot is a smeared version of the LIGO antenna pattern of Figure 1. In particular, the four minima of the LIGO sensitivity are smeared into two minima that are wider but not so deep. The figure of merit is nowhere zero, varying by a factor of approximately 4 between best (near zenith of detectors) and worst (near planes of detectors at 45° from arms) orientations of *Swift*. The all-sky average of the figure of merit—expected if *Swift* is pointed

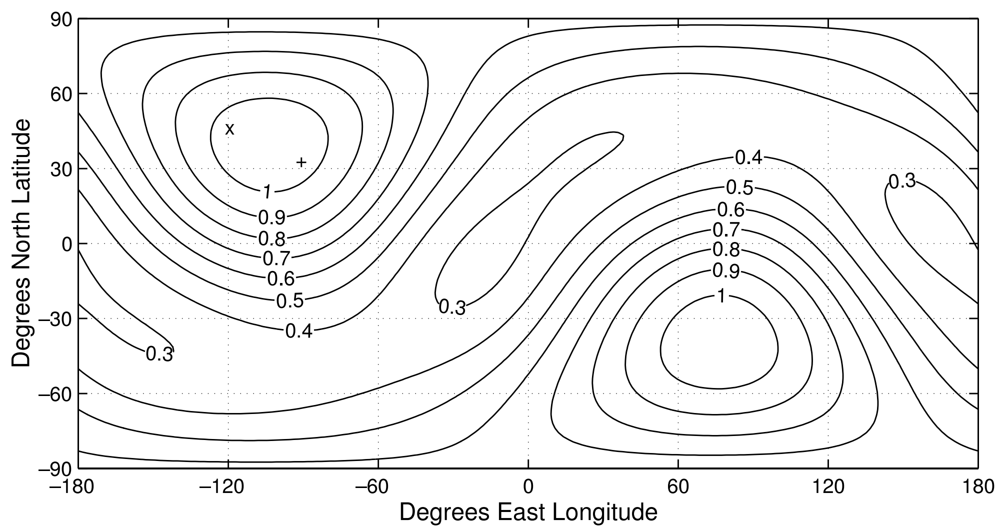

FIG. 2.—Figure of merit ζ (eq. [31]) for *Swift* pointing in Earth-based coordinates, produced by convolving the LIGO sensitivity pattern $\rho_{\rm GWB}$ (eq. [28]) with the *Swift* sensitivity function ρ_{Swift} (eq. [33]). The four minima of the LIGO sensitivity pattern of Fig. 1 are smeared into two minima that are not so deep. The figure of merit is nowhere zero, having a range of [0.25, 1.00] and an all-sky average of 0.56. The plus sign and cross mark the locations of the LLO and LHO detectors.

without reference to LIGO—is 0.56 times the maximum value. We thus conclude that the upper limit one can attain on the mean-square gravitational wave amplitude by pointing *Swift* along the optimal directions is approximately a factor of 2 better than what can be achieved by orienting the satellite without taking the LIGO detectors' orientation into account.

5. DISCUSSION

It is widely believed that gamma-ray bursts (GRBs) originate in relativistic fireballs produced by the catastrophic merger or collapse of solar-mass compact objects. Gravitational waves should be associated with these events, and their detection would permit these models to be tested, the GRB progenitor (e.g., coalescing binary or collapsing stellar core) to be identified, and the origin of the gamma rays (within the expanding relativistic fireball or at the point of impact on the interstellar medium) to be located. Even upper limits on the gravitational wave strength associated with GRBs could constrain the GRB model.

We have evaluated how the quality of an upper limit on the gravitational wave strength associated with GRB observations depends on the relative orientation of the GRB and gravitational wave detectors, with particular application to the *Swift* Burst Alert Telescope (BAT) and the LIGO gravitational-wave detectors. Setting aside other physical and science constraints on the *Swift* mission, careful choice of BAT pointing leads to an upper limit on the observed GRB population-averaged mean-square gravitational wave strength a factor of 2 lower than the upper limit resulting from pointing that does not take this science into account. This optimization does not depend on any specific model for the GRB phenomenon.

There are, of course, numerous science and technical constraints that determine the pointing profile of a satellite like *Swift*; correspondingly, even in the most optimistic case the ratio of the best attainable upper limit to the all-sky upper limit will be reduced from this factor of 2. Nevertheless, when it can be done without jeopardizing other mission objectives there is an advantage to optimizing the pointing of *Swift* to maximize the joint LIGO/*Swift* sensitivity to GRB systems. The pointing-dependent part (cf. eq. [31]) of the anticipated upper limit that can be set with these joint observations, suitably normalized, is closely related to the science enabled by joint LIGO/*Swift* observations and constitutes an excellent "figure of merit" that can be used to incorporate this objective in *Swift* mission planning.

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REFERENCES

Allen, B., & Romano, J. D. 1999, Phys. Rev. D, 59, 102001
Althouse, W. E., Hand, S. D., Jones, L. K., Lazzarini, A., & Weiss, R. 2001, Rev. Sci. Instrum., 72, 3086
Astone, P., et al. 2002, Phys. Rev. D, 66, 102002
Cavallo, G., & Rees, M. J. 1978, MNRAS, 183, 359
Cramer, H. 1999, Mathematical Methods of Statistics (Princeton: Princeton Univ. Press)
Eichler, D., et al. 1989, Nature, 340, 126
Feldman, G. J., & Cousins, R. D. 1998, Phys. Rev. D, 57, 3873
Finn, L. S., Mohanty, S. D., & Romano, J. D. 1999, Phys. Rev. D, 60, 121101
Frail, D. A., et al. 2001, ApJ, 562, L55
Goodman, J. 1986, ApJ, 308, L47
Hjorth, J., et al. 2003, Nature, 423, 847
Kobayashi, S., & Mészáros, P. 2003a, ApJ, 585, L89
———. 2003b, ApJ, 589, 861
Kobayashi, S., Piran, T., & Sari, R. 1997, ApJ, 490, 92
MacFadyen, A., & Woosley, S. 1999, ApJ, 524, 262
Mészáros, P. 2002, ARA&A, 40, 137
Mészáros, P. 2003, Nature, 423, 809
Mészáros, P., & Rees, M. J. 1993, ApJ, 405, 278
———. 1997, ApJ, 482, L29
Metzger, M. R., et al. 1997, Nature, 387, 878
Paczyński, B. 1986, ApJ, 308, L43
———. 1998, ApJ, 494, L45
Price, P. A., et al. 2003, Nature, 423, 844
Rees, M. J., & Mészáros, P. 1992, MNRAS, 258, P41
———. 1994, ApJ, 430, L93
Sari, R., & Piran, T. 1997, ApJ, 485, 270
Siegal, S., & Castellan, N. J., Jr. 1988, Nonparametric Statistics for the Behavioral Sciences (New York: McGraw-Hill)
Sigg, D. 2001, Classical Quantum Gravity, 19, 1429
Snedecor, G. W., & Cochran, W. G. 1967, Statistical Methods (Ames: Iowa State Univ. Press)
van Putten, M. H. P. M. 2004, in GRBs in the Afterglow Era: 3rd Workshop, in press (astro-ph/0301607)
Woosley, S. E. 1993, ApJ, 405, 273